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Mathematica Slovaca

DOI: 10.1515/ms-2024-0096
Math. Slovaca **74** (2024), No. 5, 1321–1342

RESIDUAL INACCURACY EXTROPY AND ITS PROPERTIES

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(*Communicated by Gejza Wimmer*)

ABSTRACT. In this paper, we introduce a novel concept of a dynamic residual inaccuracy measure based on extropy. We extend the traditional residual inaccuracy measure to its dynamic version, which allows us to measure the discrepancy between two residual life distributions. We study the properties of the proposed measure, including its discrimination principle and the proportional hazard rate model. We also investigate a characterization problem related to the extropy inaccuracy measure and propose some alternative expressions of the dynamic residual measure of inaccuracy. Furthermore, we establish upper and lower bounds and some inequalities concerning dynamic residual inaccuracy measures based on extropy. We demonstrate that the defined measure of inaccuracy is invariant under scale but not under location transformation. The given findings have important implications for statistical inference, estimation, and modeling. The proposed extropy-based dynamic residual inaccuracy measure provides a powerful tool for quantifying the discrepancy between two residual life distributions over time. At the end of the paper, we provide two non-parametric estimators for the proposed extropy measure of inaccuracy for both the non-censored (complete sample) and the right-censored scheme. The performances of these estimators are compared numerically based on their bias and MSE.

1. Introduction

The concept of entropy has fascinated scientists for decades, with its roots tracing back to the pioneering work of physicist Boltzmann [14]. Initially proposed as a measure of chaos in physical systems, entropy has since evolved to become a fundamental concept in information theory, with Shannon's [22] groundbreaking work providing a mathematical definition of information, famously known as Shannon entropy. Since then, researchers from various disciplines have developed numerous entropy and information indices, ranging from parametric to nonparametric points of view. One such development, Kerridge's [11] inaccuracy measure, stands out as a non-parametric generalization of Shannon entropy. This powerful tool has found widespread applications in fields such as statistics, machine learning, and data analysis.

Given two absolutely continuous random variables X and Y with cumulative density functions (CDFs) $F(x)$ and $G(x)$, and probability density function (PDFs) $f(x)$ and $g(x)$, if $F(x)$ is the actual distribution corresponding to the observations and $G(x)$ is the distribution assigned by the experimenter, then the Kerridge inaccuracy measure of X and Y (also known as Fraser information

2020 Mathematics Subject Classification: Primary 62F10, 62N05.
Keywords: Extropy, dynamic measure of inaccuracy, proportional hazard rate model, residual lifetime.
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