



# On weighted cumulative residual extropy: characterization, estimation and testing

M. Hashempour<sup>a</sup>, M. R. Kazemi <sup>b</sup> and S. Tahmasebi <sup>c</sup>

<sup>a</sup>Department of Statistics, University of Hormozgan, Bandar Abbas, Iran; <sup>b</sup>Department of Statistics, Faculty of Science, Fasa University, Fasa, Iran; <sup>c</sup>Department of Statistics, Persian Gulf University, Bushehr, Iran

## ABSTRACT

This paper considers a new generalization of cumulative residual extropy (CRJ) introduced by Jahanshahi et al. [On cumulative residual extropy. *Probab Eng Inf Sci*. 2019. DOI:10.1017/S0269964819000196], called weighted cumulative residual extropy (WCRJ). This paper studies some properties of WCRJ of continuous lifetime distributions. Several results including various bounds, inequalities, and effects of linear transformations are obtained. Conditional WCRJ and some of its properties are discussed. Related studies of survival analysis are covered. Also, we propose an empirical version of the WCRJ to estimate this measure of uncertainty. Based on the asymptotic distribution of empirical WCRJ, a new test statistic is given for testing the equality of two cumulative distribution functions. The power of the proposed test statistic is compared to other traditional and new competing approaches. Some simulations are carried out to show how this newly proposed method is more powerful than the others for moderate to large sample sizes.

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## 1. Introduction

Shannon [1] introduced the concept of entropy which is widely used in the fields of physics, information theory, economics, probability, statistics, communication theory, and so forth. In recent years, there has been a great interest in the measurement of the uncertainty of probability distributions. In information theory, entropy is a measure of uncertainty associated with a random variable. Shannon entropy represents the absolute limit on the best possible lossless compression of any communication. For more study, see [2]. Suppose that  $X$  is a non-negative random variable with a continuous cumulative distribution function (cdf)  $F$ , a survival function (sf)  $\bar{F}(x) = 1 - F(x)$  and a probability density function (pdf)  $f$ . The traditional measure of uncertainty is differential entropy, commonly termed by Shannon [1] information measure, defined as

$$H(X) = -E(\log f(X)) = -\int_{-\infty}^{+\infty} f(x) \log f(x) dx,$$