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Abstract

The aim of this paper is to make inference about a general class of time series models including fractional Brownian motion. The spectral of these processes is supported on lines not parallel to the diagonal $T_1(x) = x$, $T_j(x) = \alpha_j x \pm \beta_j$, $j = 2, \ldots, m$, in spectral square $[0, 2\pi) \times [0, 2\pi)$, and this class includes stationary, cyclostationary, almost cyclostationary time series and specially fractional Brownian motions. First, the periodogram of these processes is defined and auxiliary operator is applied to explore the distribution of the periodogram. Then the asymptotical estimation for the spectral density function is proposed and asymptotical Wishart function is found. Finally, the validity of the theoretical results is studied using simulated data sets.

Keywords: Time Series; Fractional Brownian Motion; Spectral Analysis; Discrete Fourier Transform; Periodogram.

1. INTRODUCTION

Stationary processes are frequently used in time series modeling, specially in stochastic signals. These processes have spectra on the main diagonal, $T_1(x) = x$ in the square frequency plane, $[0, 2\pi)^2$. In 1970, Hannan[1] studied wide range of statistical application of stationary time series. Later Brockwell and Davis[2] introduced some basic ideas of time series analysis and stochastic processes. Then stationary time series were widely discussed by many scientists.[3–12] Although these processes can be nicely modeled by many time series, the stochastic rhythm of many processes is periodic. In these cases, cyclostationary (periodically correlated) and almost cyclostationary time series are alternatives to explain the stochastic rhythm in time series. As discussed by Gladyshev,[13] the mean, auto-covariance and auto-correlation functions of cyclostationary time series are periodic. The spectra of cyclostationary processes with m cyclic period are supported on lines (with equal space) parallel to the main diagonal $T_j(x) = x \pm \frac{2\pi j}{m}$, $j = 0, 2, \ldots, m-1$ in the square frequency plane $[0, 2\pi)^2$. The mean, auto-covariance and auto-correlation functions of almost cyclostationary processes are also almost periodic. The spectra of these processes are supported on lines parallel to the diagonal $T_1(x)$, $T_j(x) = x \pm \alpha_j$, $j = 2, \ldots, m$ in the square frequency plane $[0, 2\pi)^2$ and include stationary and cyclostationary classes. References 14–38 considered the cyclostationary and almost cyclostationary processes. To study more properties of time varying systems, series like predefined-time convergence in fractional-order systems, approximation of the evolutionary Hamilton–Jacobi–Bellman equations (HJB equation) and involves a weakly coupled discrete system of parabolic quasi-variational inequalities (PQVs) were investigated, see Refs. 39–41.

As can be seen, the spectral measures of these two classes of processes have supported on the lines parallel to main diagonal. Although these processes can be nicely modeled by many