entropy

MDPI

Article

Weighted Cumulative Past Extropy and Its Inference

Mohammad Reza Kazemi [1], Majid Hashempour [2] and Maria Longobardi [3,*]

[1] Department of Statistics, Faculty of Science, Fasa University, Fasa IS74, Iran
[2] Department of Statistics, University of Hormozgan, Bandar Abbas 79177, Iran
[3] Dipartimento di Biologia, Università degli Studi di Napoli Federico II, 80126 Naples, Italy
* Correspondence: maria.longobardi@unina.it

Abstract: This paper introduces and studies a new generalization of cumulative past extropy called weighted cumulative past extropy (WCPJ) for continuous random variables. We explore the following: if the WCPJs of the last order statistic are equal for two distributions, then these two distributions will be equal. We examine some properties of the WCPJ, and a number of inequalities involving bounds for WCPJ are obtained. Studies related to reliability theory are discussed. Finally, the empirical version of the WCPJ is considered, and a test statistic is proposed. The critical cutoff points of the test statistic are computed numerically. Then, the power of this test is compared to a number of alternative approaches. In some situations, its power is superior to the rest, and in some other settings, it is somewhat weaker than the others. The simulation study shows that the use of this test statistic can be satisfactory with due attention to its simple form and the rich information content behind it.

Keywords: weighted cumulative past extropy; reliability theory; empirical extropy; goodness of fit test

MSC: 62B10; 62N05

check for updates

Citation: Kazemi, M.R.; Hashempour, M.; Longobardi, M. Weighted Cumulative Past Extropy and Its Inference. *Entropy* **2022**, *24*, 1444. https://doi.org/10.3390/e24101444

Academic Editor: Joaquín Abellán

Received: 14 September 2022
Accepted: 8 October 2022
Published: 11 October 2022

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.

1. Introduction

In recent years, there has been strong interest in the measurement of the uncertainty of probability distributions, which is called entropy. The probabilistic concept of entropy was developed by [1]. For an absolutely continuous random variable X, the Shannon entropy is defined as

$$H(X) = -E(\log f(X)) = -\int_{-\infty}^{+\infty} f(x)\log f(x)dx,$$

where " log " means the natural logarithm, and $f(x)$ is the probability density function (pdf) of a random variable X. Several applications of entropy in information theory, economics, communication theory, and physics are well developed in the literature, (see Cover and Thomas, [2]). Belis and Guiasu [3] and Guiasu [4] considered a weighted entropy measure as

$$H^w(X) = -E(X\log f(X)) = -\int_{-\infty}^{+\infty} xf(x)\log f(x)dx, \tag{1}$$

where by assigning greater importance to larger values of X, the weight x in (1) emphasizes the occurrence of the event $X = x$. Reference [5] stated the necessity of the existence of the weighted measures of uncertainty. In the Shanon entropy $H(X)$, only the pdf of the random variable X is regarded. Moreover, it is known that this information measure is shift-independent, in the sense that the information content of a random variable X is equal to that of $X + b$. Indeed, some applied fields such as neurobiology do not tend to deal with shift-independent but shift-dependent. Further research was conducted to generalize the concept of entropy, for example, by replacing the pdf $f(x)$ with the survival function $\bar{F}(x)$, [6] introduced the cumulative residual entropy (CRE) as