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# A computational technique to classify several fractional Brownian motion processes

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ABSTRACT

In this paper, for the first time, the classification of several fractional Brownian motion time series is considered. For this purpose, fuzzy clustering technique is applied and Brownian motion processes are classified. The applicability of the given approach is explored using simulated and a real COVID-19 dataset.



## 1. Introduction

In practice, numerous time series models can be applied to model an observed time series dataset. But there are some important issues:

- Which models are the best?
- Are there significant differences between these fitted models?

To compare some models fitted on a dataset, researchers commonly employ various selection criterion such as coefficient of determination ($R^2$), mean absolute error (MAE), mean square error (MSE), and information criterion such as Akaike (AIC) and Bayesian (BIC). The models with smaller values of MAE, MSE, BIC and AIC, or larger values of $R^2$ are selected as the best models. For example, Zarei and Mahmoudi [1] and Zarei et al. [2,3] studied different models fitted on drought index in some hydrological stations and compared and ranked them based on different indices. Based on these indices, we can only rank the fitted models. But the second question remains still unsolved. To response this question, there are huge studies (for example, see De Souza and Thomson [4]; Coates and Diggle [5]; Potscher and Reschenhofer [6]; Piccolo et al. [7]; Diggle and Fisher [8]; Cowpertwait and Cox [9]; Dargahi-Noubary [10]; Macchiato et al. [11]; Maharaj [12]; Diggle and al Wasel [13]; Kakizawa et al. [14]; Timmer et al. [15]; Maharaj [16–18]; Alonso et al. [19,20]; Pattarin et al. [21]; Fruhwirth-Schnatter and Kaufmann [22]; Alonso et al. [23]; Caiado et al. [24]; Eichler [25]; Fokianos and Savvides [26]; Caiado et al. [27]; Dette and Paparoditis [28]; Dette et al. [29]; Dette and Hildebrandt [30]; Jentsch [31]; Jentsch and Pauly [32]; Salcedo et al. [33]; Jentsch and Pauly [34]; Triacca [35]; Mahmoudi et al. [36–39]). These researches have considered the following situations:

- Comparison between two or several dependent or independent stationary processes
- Classification of several dependent or independent stationary processes
- Comparison between two or several dependent or independent cyclostationary processes
- Classification of several dependent or independent cyclostationary processes

In this study, for the first time, the classification of several fractional Brownian motion processes is considered. For this purpose, fuzzy clustering technique is applied and Brownian motion time series are classified. For this purpose, based on the fitted fractional Brownian motion time series, the estimated values for observed dataset are computed. Then, the fuzzy clustering technique is applied and the fitted fractional Brownian motion proceses are clustered. The applicability of the given method is explored using simulated and real datasets.

## 2. Method

Suppose $B_H(t)$ is a fractional Brownian motion (fBm, in abbreviation) with Hurst index $H \in (0,1)$, defined by

$$B_H(t) = \frac{1}{\Gamma\left(H+\frac{1}{2}\right)} \int_0^t (t-s)^{H-\frac{1}{2}} dB(s),$$

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