entropy

MDPI

Article

Fractional Deng Entropy and Extropy and Some Applications

Mohammad Reza Kazemi [1], Saeid Tahmasebi [2], Francesco Buono [3] and Maria Longobardi [4,*]

[1] Department of Statistics, Faculty of Science, Fasa University, Fasa 746-168-6688, Iran; kazemi@fasau.ac.ir
[2] Department of Statistics, Persian Gulf University, Bushehr 751-691-3817, Iran; tahmasebi@pgu.ac.ir
[3] Dipartimento di Matematica e Applicazioni "Renato Caccioppoli", Università degli Studi di Napoli Federico II, 80138 Naples, Italy; francesco.buono3@unina.it
[4] Dipartimento di Biologia, Università degli Studi di Napoli Federico II, 80138 Naples, Italy
* Correspondence: maria.longobardi@unina.it

Abstract: Deng entropy and extropy are two measures useful in the Dempster–Shafer evidence theory (DST) to study uncertainty, following the idea that extropy is the dual concept of entropy. In this paper, we present their fractional versions named fractional Deng entropy and extropy and compare them to other measures in the framework of DST. Here, we study the maximum for both of them and give several examples. Finally, we analyze a problem of classification in pattern recognition in order to highlight the importance of these new measures.

Keywords: measures of uncertainty; fractional entropy; Deng entropy and extropy; classification and discrimination

MSC: 62H30; 62B10; 94A17

check for updates

Citation: Kazemi, M.R.; Tahmasebi, S.; Buono, F.; Longobardi, M. Fractional Deng Entropy and Extropy and Some Applications. *Entropy* **2021**, *23*, 623. https://doi.org/10.3390/e23050623

Academic Editor: Yong Deng

Received: 29 April 2021
Accepted: 14 May 2021
Published: 17 May 2021

1. Introduction

The concept of entropy as a measure of uncertainty was first introduced by Shannon [1], and since then, it has been used in the field of information theory, image and signal processing and economics. Let X be a discrete random variable with probability mass function vector $\underline{p} = (p_1, \dots, p_n)$. The Shannon entropy of X is defined as follows

$$H(X) = H(\underline{p}) = -\sum_{i=1}^{n} p_i \log p_i, \tag{1}$$

where $\log(\cdot)$ stands for the natural logarithm with the convention $0 \log 0 = 0$. Recently, the dual measure of entropy has become widespread. It is known as extropy and was defined for a discrete random variable X by Lad et al. [2] as

$$J(X) = J(\underline{p}) = -\sum_{i=1}^{n} (1 - p_i) \log(1 - p_i), \tag{2}$$

and since then, as the Shannon entropy, it has been studied in several contexts and in its differential version [3–6].

The generalization of Shannon entropy to various fields is always of great interest. Ubriaco [7] defined a new entropy based on fractional calculus as follows:

$$S_q(X) = S_q(\underline{p}) = \sum_{i=1}^{n} p_i [-\log p_i]^q, \quad 0 < q \leq 1. \tag{3}$$

The fractional entropy is concave, positive and non-additive. Moreover, for $q = 1$, the fractional entropy reduces to the Shannon entropy. From a physical sense, it also satisfies Lesche and thermodynamic stability.