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Results on Varextropy Measure of Random Variables

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Abstract: In 2015, Lad, Sanfilippo and Agrò proposed an alternative measure of uncertainty dual to the entropy known as extropy. This paper provides some results on a dispersion measure of extropy of random variables which is called varextropy and studies several properties of this concept. Especially, the varextropy measure of residual and past lifetimes, order statistics, record values and proportional hazard rate models are discussed. Moreover, the conditional varextropy is considered and some properties of this measure are studied. Finally, a new stochastic comparison method, named varextropy ordering, is introduced and some of its properties are presented.

Keywords: extropy; uncertainty; varextropy; residual lifetime

MSC: 62B10; 62G30

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1. Introduction

In the context of information theory, entropy was first proposed by Clausius. He used the concept of entropy to quantitatively express the second law of thermodynamics, which opened up a new path for the development of thermodynamics [1]. This concept was continued by Shannon [2] and since then it has been used in several fields, such as image and signal processing and economics. Let X be an absolutely continuous random variable with probability density function (pdf) $f(x)$; the differential entropy is a measure of uncertainty, and is defined by

$$H(X) = -\int_{-\infty}^{+\infty} f(x) \log f(x) \mathrm{d}x,$$

where $\log(\cdot)$ stands for the natural logarithm with the convention $0 \log 0 = 0$. Song [3] introduced the concept of varentropy (VE) as an excellent alternative for the kurtosis measure. In fact, the VE can be used to compare the heavy-tailed distributions instead of kurtosis measure. Liu [4] studied some properties of VE under the concept of information volatility. Fradelizi et al. [5] obtained an optimal varentropy bound for log-concave distributions. The varentropy of a random variable X is defined as

$$VH(X) = Var(-\log f(X)). \tag{1}$$

Varentropy measures the variability in the information content of X. Recently, Di Crescenzo and Paolillo [6] studied the varentropy for residual lifetime. Maadani et al. [7] introduced a method for calculating this measure for the i-th order statistic. An alternative measure of uncertainty, known as extropy, was proposed by Lad et al. [8]. For an absolutely continuous random variable X with pdf $f(x)$, the extropy is defined as